International Journal of Theoretical Physics, Vol. 36, No. 11, 1997

On the Quantum Electrodynamics of Moving Bodies

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Received June 25, 1997

A new synthesis of the principles of relativity and quantum mechanics is developed by replacing the Poincaré group for the de Sitter one. The new relativistic quantum mechanics is an indefinite-mass theory which is reduced to the standard theory on the mass shell. The charge conjugation acquires a geometrical meaning and the Stueckelberg interpretation for antiparticles naturally arises in the formalism. So the idea of the Dirac sea in the second-quantized formalism proves to be superfluous. The off-shell theory is free from ultraviolet divergences, which only appear in the process of mass-shell reduction.

The advent of quantum theory led to the hope of reformulating electrodynamics free from anomalies. However, divergences were smoothed, but not completely erased by quantization. Such a disappointment was considered as a serious trouble for the physics of that time and the progress in the area was delayed for two decades. After the great advances achieved by the end of the 1950s, a new generation of physicists "have learned how to peacefully coexist with the alarming divergences of the old fashioned theory, but these infinities are still with us, even though deeply buried in the formalism" (Roman, 1969). Due to this fact some workers in the field tried to start again from the beginning, formulating the so-called axiomatic quantum field theory. Their dissatisfaction was clearly summarized in the statement of Streater and Wightman (1964), "...the quantum theory of fields never reached a stage where one could say with confidence that it was free from internal contradictions—nor the converse." Unfortunately, as Rohrlich (in Jauch and Rohrlich, 1976), has pointed out, this route does not fulfill all aspirations: "We now have a much deeper mathematical understanding of quantum electrodynamics,

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0020-7748/97/1100-2391$12.50/0

especially due to the work of axiomatic field theorists; but we have still not solved the basic problem of formulating the theory in a clean mathematical way, not even with all the complicated and highly sophisticated limiting procedures presently used to justify the results of a naive renormalization theory in simpler quantum field theories and in lower dimensionality. The hopes and aspirations indicated in the outlook of twenty years ago remain valid today."

A renovating spirit was present in the more recent movement of string theorists who decided to change some basic principles. As a consequence, string models have nonlocal interactions which provide a way to avoid the ultraviolet divergences from the beginning. However, the price paid for this desirable requirement is too high: we have lost the extraordinary power of the calculus and predictability of quantum field theory. This is the reason why some theoretical physicists became conservative and, in a radical change, tried to justify "the unreasonable effectiveness of quantum field theory" (Jackiw, 1996), arguing that the phenomenologically desirable results are provided by ultraviolet divergences. As in the standard theoretical framework, anomalies, such as the chiral one, come from the gauge noninvariance of the infinite negative-energy sea. It is argued that "we must assign physical reality to this infinite negative-energy sea" (Jackiw, 1986). We see such a philosophical position as a new attempt at rescuing the theory of the "ether." Alternatively, Weinberg (1997) has dealt with the present difficulties for quantizing gravity by reformulating the problem, holding that the standard model and general relativity are the leading terms in effective field theories, and so he disregards the problem of renormalizability, which is only proper to a still unknown fundamental theory (perhaps a string model).

On the contrary, the creators of the quantum field theory, such as Dirac (1968), held a less conservative viewpoint:

> Nowadays, most of the theoretical physicists are satisfied with this situation, but I am not. I think that theoretical physicists have taken a wrong way with those new facts and we would not be pleased with this situation. We must understand that we are in front of something wrong radically discarding the infinities from our equations; here we need to respect the basic laws of the logics. Thinking about this point could send us to an important advance. QED is the branch of theoretical physics about we know more, and presumably we have to put it in order until we can make a fundamental progress in other field theories, although these theories continue developing under experimental basis.

In this work we develop the foundations of a new synthesis of the principles of relativity and quantum mechanics. Following Dirac's advice, we only propose to reformulate QED. As our purpose is humbler than that of the string program (conceived as the theory of everything), the change in the basic principles is also less radical: essentially we propose to substitute once more the standard group of external symmetries, i.e., the Poincaré group for the de Sitter one. It is ironic that, approaching the end of this century, nine decades after Einstein did the same with the Galilei group, we can motivate the new program, rephrasing Einstein's words (1905):

It is known that Dirac's quantum electrodynamics—as usually understood at the present time—leads to asymmetries and inconsistencies which do not appear to be inherent in the phenomena. Take, for example, the description of a pair creation in an external electromagnetic field. The observable phenomenon here always involves finite measurable quantities and does not make any distinction between electron and positron, whereas the customary view draws a sharp distinction between the two particles. While the electron is interpreted as a positive-energy state of the Dirac equation, the positron is interpreted as a hole or absence of a negative-energy state in the Dirac sea.[2] This sea of infinite electrons, which fills all the negative-energy states of the Dirac equation, is responsible for ultraviolet divergences in the effective action used for describing such phenomena.[3] Moreover, from the standpoint of general relativity, the zero-point energy of the electromagnetic field also seems unsatisfactory, since a divergent vacuum stress tensor would imply, via the Einstein field equations, an infinite curvature for the universe corresponding to an infinite cosmological constant, which cannot be removed simply by performing some sort of transfinite shift of the energy scale.

Examples of this sort, together with the unsuccessful attempts at quantizing gravity through these methods, suggest that the phenomena of electrodynamics as well as of gravity at a quantum level possess no properties corresponding to the quantum field notion of the vacuum.[4] They rather suggest that a different route must be taken in order to accommodate the principles of relativity at the quantum level. From our point of view the main difficulty lies in the different roles and interpretations of "time" in both theories. In fact, while quantum mechanics privileges an absolute parameter that labels the evolution of the system, the theory of relativity stresses the relative character of the temporal coordinate. Therefore the first concept of time

[2] The asymmetry in the description is more evident from the historical point of view. In fact the holes were originally interpreted by Dirac (1930) as protons, who thought that he could explain the mass differences by means of the interaction of the electrons of the sea.

[3] This is analogous to the case of chiral anomaly discused above, and it results especially clearly from the Weisskopf derivation of the Heisenberg–Euler Lagrangian (Lifshitz and Pitaevskii, 1971). In Section 2 we discuss the proper time approach to this effective Lagrangian, in which it becomes clear that divergences appear in the transition from the off-shell theory to the mass shell.

[4] As we will see, we do not discard many-"particle" formalisms (we find it more appropriate to call them many-charge formalisms) nor the notion of field. We only attack the choice of the vacua in standard quantum field theory to implement the charge conjugation symmetry.

should have the properties of a c-number, while the second should be an operator due to the mixing character of the Lorentz transformations. Thus this dual role of time poses a problem in relativistic quantum mechanics at a first-quantized level. The standard solution to this dilemma is to give up this vessel and plunge into the sea of quantum field theory, relegating the role of space-time coordinates to be simple parameters of the theory. Unfortunately, this mathematical artifact is achieved by means of a choice of vacuum compatible with the idea of the Dirac sea, which actually just swept the problem under the rug. This fact suggests that such a dual role of time demands the introduction of two different concepts for playing two different roles. In other words, we propose that the unification of quantum principles with the theory of relativity requires the introduction of an additional label to describe the events,[5] increasing in this way the dimension of the space-time manifold (Aparicio *et al.*, 1995; Gaioli and Garcia Alvarez, 1995a, 1996). We will raise this conjecture to the status of a postulate, and also introduce another postulate, namely, laws of physics in our five-dimensional space-time obey the principles of the special theory of relativity. These two postulates suffice for the attainment of a simple and consistent theory of quantum electrodynamics, based on Dirac's theory in a higher dimension. The introduction of a "Dirac sea" will prove to be superfluous inasmuch as the view to be developed here will not require ordinary time to be the parameter which labels the quantum evolution.

1. KINEMATICAL PART

Nowadays, theoretical physicists seem to be more focused on internal symmetries than on external ones, in search of a grand unified gauge theory. However, in the 1960s a great effort was made on unifying both symmetries, enlarging the Poincaré group. So for different motivations the simplest extensions of the Poincaré group, such as the five-dimensional Galilei group, the de Sitter group, and conformal group, began to be studied, constituting the antecedents of our program.[6] However, the idea of enlarging the dimension of space-time to take into account particle–antiparticle symmetries is an older, fascinating idea. Perhaps the first antecedent can be found in the works of Hinton, who built a model of electricity associating positive and negative

[5] Formulations of relativistic quantum mechanics with an invariant evolution parameter were discussed in the past. According to the external group of symmetry, they can be classified as five-dimensional Galilei-invariant formulations (Aghassi *et al.*, 1970a; Horwitz and Piron, 1973; Fanchi, 1993) and de Sitter ones. See Aparicio *et al.* (1995a, b) for a critical review.

[6] In connection with this work see Castell (1966), De Vos and Hilgevoord (1967), Aghassi *et al.* (1970a, b), Johnson (1969, 1971), and Johnson and Chang (1971).

charges with right- and left-handed helixes in higher dimensional spaces. Curiously, this prerelativistic model developed in 1888 has an extraordinary parallelism with the theory of Klein (Gardner, 1994). In Section 2 we discuss these ideas through a generalization of the Schrödinger *Zitterbewegung* to four dimensions (Barut and Zanghi, 1984; Barut and Thacker, 1985; Gaioli and Garcia Alvarez, 1996), which is related to the Stueckelberg (1941) and Wheeler and Feynman (1948, 1949, 1950, 1951; Schweber, 1986; Nambu, 1950) interpretation of antiparticles. But along this route, the concept of time must be revisited.

Time in physics is not an *a priori* concept in the Newtonian sense, but enters as a basic concept used to describe the laws of nature. The history of science shows us that physics always adapts and modifies this concept in order to simplify the laws. Then, from this point of view, there is no point to the question of why the universe has five dimensions and not four. The important thing is that there is a set of phenomena which can be described in a more simple and symmetrical way if we use two times instead of one. The purpose of this work is to demonstrate that this is the case for QED.

We begin by considering a five-dimensional manifold as the space-time arena in which such phenomena occur. According to the first postulate, each event in our description has associated a point P of the space-time determined by coordinates $x^A = (x^\mu, x^5)$ $(A = 0, 1, 2, 3, 5)$, i.e., $P = P(x^A)$, which will be called a super-event. From the second postulate the space-time is endowed with a super-Minkowskian metric $g^{AB} = \mathrm{diag}(+, -, -, -, -)$, so the square of the super-arc element dS reads

$$dS^2 = g^{AB}\, dx_A\, dx_B = g^{\mu\nu}\, dx_\mu\, dx_\nu - (dx^5)^2 \tag{1}$$

Any linear transformation of coordinates $x^{A'} = L^A_B x^B + C^A$ which leaves dS^2 invariant will be referred to as a coordinate transformation between two super-inertial systems. The super-Poincaré group of such a transformation is the well-known inhomogeneous de Sitter group. The other implicit assumption is that all physical laws adopt the same form in all super-inertial frames, that is to say, they are de Sitter-covariant.

We do not analyze here all the potentialities of such a description, but our intention is to use this new framework to reformulate the physics associated with the Poincaré invariance free from inconsistencies. Keeping this in mind, let us restrict ourselves to the subset of linear transformations

$$x^{\mu'} = L^\mu_\nu x^\nu + C^\mu \tag{2}$$

$$x^{5'} = x^5 + C^5 \tag{3}$$

which leaves the square of the standard arc element, $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$, invariant, maintaining the fifth coordinate x^5 as a Poincaré-invariant parame-

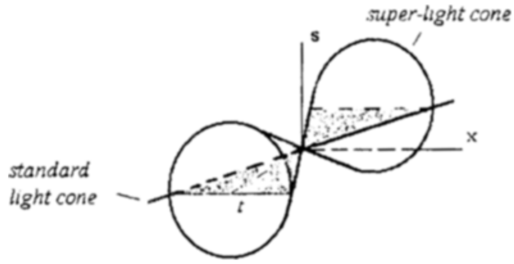

Fig. 1. Super and standard light cones.

ter. This means that we are going to describe the superevents posed in a given super-frame, forbidding boosts and rotations between x^5 and any of the space-time coordinates. In this case such an evolution parameter works as a Newtonian time in each super-frame and introduces an absolute notion of simultaneity and retarded causality associated to it. The fifth coordinate x^5 is arbitrary in principle; however, from equation (1) we see that for the particular case of motions on the super-light cone ($dS = 0$) the coordinate x^5 is reduced to s. We restrict our analysis of QED to this case. In Fig. 1 we show the super-light cone and its four-dimensional projection. Note that while a super-world line lies on the super-light cone, its space-time projection lies inside the standard light cone.

At this point one could ask what we have gained with such a description. The immediate answer is that this description now has an invariant evolution parameter at the classical level, preparing the way for a description at the quantum level that avoids the lack of explicit covariance of the standard canonical formalism. What is not so evident is that it is a natural framework for introducing the notion of antiparticles. Moreover, as we show in Section 2, the notion of retarded causality in x^5 for super-particles naturally leads to the standard quantum field-theoretic boundary conditions for the Green functions on the mass shell. That is, particles go forward and antiparticles go backward in the coordinate time x^0.[7]

Let us consider the world-line of a super-event in a given super-frame. The Poincaré invariance suggests that we parametrize this curve with x^5, i.e., project the super-world-line in a hyperplane x^5 = const (the standard space-time). Thus, at any point of the projected curve (a standard world-line), the four-velocity $dx^\mu/dx^5 = (dx^0/dx^5, dx^0/dx^5)$ has a new key ingredient with respect to the noncovariant description which takes the coordinate x^0 as the evolution parameter, namely the rate dx^0/dx^5. This new degree of freedom

[7] This formalism allows us to reformulate the "localization problem" (Kálnay, 1971) by following charge "trajectories" instead of particles ones. Moreover, the recognition that this strange notion of x^0-causality is the only one compatible with the requirements of relativistic quantum mechanics enables one to eliminate Hegerfeldt's (1974) paradox.

allows us to introduce the concept of antiparticle just at the classical level. Generalizing the ideas of Stueckelberg (1941, 1942; Feynman, 1948), we call super-particles and super-antiparticles those states for which dx^0/dx^5 is positive and negative, respectively. Therefore, for causal propagation ($dx^5 > 0$), while the super-particles propagate forward in time, the super-antiparticles propagate backward in coordinate time. Notice that for $dx^5 = 0$ we cannot distinguish the two concepts.[8] This is the case of the photon in the standard framework, in which we identify the fifth coordinate with the classical proper time. We could expect at first glance that the evolution in x^5 also interchanges particle and antiparticle states. Nevertheless, as we will see below, for the standard electromagnetic interactions this interchange is classically forbidden and only possible at the quantum level as a consequence of the uncertainty principle.

2. ELECTRODYNAMIC PART

From a dynamical point of view the main difference between the Poincaré and the de Sitter groups is that for the second group the operator $p_\mu p^\mu$ is no longer a Casimir operator. The states of the new theory are off the mass shell, $p_\mu p^\mu = m^2$. They are on the super-mass-shell hyperboloid

$$p_A p^A = M^2 \tag{4}$$

where M is a super-mass parameter. We are interested in the study of null-super-mass states because in the classical limit their motion is superluminal and, as we discuss in the kinematical part, we can identify the five-coordinate x^5 with the proper time s. So, let us begin by considering the wave equation satisfied by the non-super-massive ($M = 0$) spin-1/2 irreducible representation of the de Sitter group Ψ,

$$\Gamma^A i\partial_A \Psi = 0 \tag{5}$$

where $\Gamma^\mu = \gamma^5\gamma^\mu$, $\Gamma^5 = \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$, satisfy the Dirac algebra

$$\Gamma^A\Gamma^B + \Gamma^B\Gamma^A = 2g^{AB} \tag{6}$$

Multiplying on the left by γ^5, we can rewrite (5) in the Hamiltonian form

$$-i\frac{\partial\Psi}{\partial s} = \gamma^\mu i\partial_\mu\Psi \tag{7}$$

[8] Also note that this notion is super-frame-dependent, i.e., a state registered as a super-particle from a super-inertial system can be registered as a super-antiparticle from another super-inertial system. The same thing happens with the notion of simultaneity associated to the coordinate x^5, which loses its invariant character under the full de Sitter group transformations.

where we have identified x^5 with s (Aparicio *et al.*, 1995a). Equation (7) was originally introduced by Feynman in 1948 in his dissertation at the Pocono Conference.[9] This is a Schrödinger equation in the invariant parameter s for the evolution of states off the mass shell. The mass-shell condition is satisfied by stationary states, $\Psi(x^\mu, s) = \psi_m(x^\mu)e^{ims}$, solutions of the Dirac equation[10]

$$\gamma^\mu i\partial_\mu\psi_m = m\psi_m \tag{8}$$

The Feynman equation minimally coupled to an external electromagnetic field is given by

$$-i\,\frac{\partial\Psi(x, s)}{\partial s} = \gamma^\mu(i\partial_\mu - eA_\mu)\Psi(x, s) \tag{9}$$

where A_μ is the electromagnetic potential.

The key idea of Feynman (1950, 1951; Schweber, 1986) was that by Fourier transforming in s any solution $\Psi(x, s)$ of (9), a solution $\psi_m(x)$ of the corresponding Dirac equation

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi_m(x) = 0 \tag{10}$$

can be obtained, namely

$$\psi_m(x) = \int_{-\infty}^{+\infty} \Psi(x, s)e^{-ims}\,ds \tag{11}$$

Hence the Fourier transform of the retarded Green function $G(x, x', s)$ of equation, (9)

$$\left[\gamma^\mu(i\partial_\mu - eA_\mu) - i\,\frac{\partial}{\partial s}\right]G(x, x', s) = \delta(x, x')\delta(s) \tag{12}$$

with $G(x, x', s) = 0$, for $s \leq 0$, enables one to derive the corresponding mass-shell Green function $G_m(x, x')$, i.e.,

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - m]G_m(x, x') = \delta(x, x') \tag{13}$$

From the path integral point of view the retarded condition for the propagator $G(x, x', s)$ means that all the classical paths go forward in time ($ds > 0$), so the on-shell positive (negative) kinetic energy states must go forward (backward) in coordinate time, since in the classical limit (neglecting spin

[9] Feynman introduced (7) in a formal way and did not discuss its geometrical meaning. He could not solve Dirac's doubts about the unitarity of the theory either. For a nice account of these anecdotes, see the review paper of Schweber (1986).

[10] The Dirac equation can be consistently introduced from first principles at a first-quantized level by interpreting antiparticles as negative-energy states going backward in x^0-time (Gaioli and Garcia Alvarez, 1995b).

effects) we have $dx^0/ds = \pm 1/\sqrt{1 - v^2}$. This fact determines the well-known boundary conditions for $G_m(x, x')$ (Feynman, 1949).

Moreover, if in the Fourier transformation

$$G_m(x, x') = \int_0^{+\infty} G(x, x', s)e^{-ims}\, ds \tag{14}$$

for the on-shell retarded Green function

$$G(x, x', s) = -i\theta(s)\langle x|e^{i\gamma^\mu\pi^\mu s}|x'\rangle \tag{15}$$

the Schwinger formal identity

$$i/(a + i\epsilon) = \int_0^{\infty} \exp[is(a + i\epsilon)]\, ds \tag{16}$$

is used for $a = \gamma^\mu\pi_\mu - m$, one immediately sees that such retarded boundary condition for $G(x, x', s)$ naturally leads to the Feynman $i\epsilon$ prescription for avoiding the poles in the on-shell Green function

$$G_m(x, x') = \left\langle x\left|\frac{1}{\gamma^\mu\pi_\mu - m + i\epsilon}\right|x'\right\rangle$$

This formal trick allowed Feynman to discuss external field problems of QED at a first-quantized level.

Let us take these formal tools further in order to understand their physical grounds. In this formalism the state space is endowed with an indefinite Hermitian form (Aparicio *et al.*, 1995a, b)

$$\langle\Psi|\Phi\rangle = \int d^4x\, \overline{\Psi}(x)\Phi(x) \tag{17}$$

in which the covariant Hamiltonian or mass operator $\mathcal{H} = \gamma^\mu i\partial_\mu$ is self-adjoint and the evolution operator $e^{i\mathcal{H}s}$ is unitary. It can be proved (Gaioli and Garcia Alvarez, 1996) that at a semiclassical level

$$\text{sign}[\overline{\Psi}(x, s)\Psi(x, s)] = \text{sign}\,\frac{dx^0}{ds} \tag{18}$$

that is, super-particles and super-antiparticles states have positive and negative norm, respectively. This is the root of the indefinite character of the "inner product." Frequently this fact is considered as an anomaly of the theory, because is not possible to straightforwardly apply the standard probabilistic interpretation. In fact this is one of the reasons why Dirac originally rejected the Klein–Gordon equation.[11] But as was shown by Feshbach and Villars (1958) that the indefinite metric character of the Klein–Gordon theory can be reinterpreted in the framework of the theory of a charge. This is the interpretation we adopt in this work.

We have defined super-particles and super-antiparticles according to the Stueckelberg interpretation in the kinematical part. Let us now show that it is consistent with the more familiar notion based on charge conjugation. To do this let us note that the operation that conjugates the charge in equation (9) is (Hannibal, 1991, 1994; Gaioli and Garcia Alvarez, 1995a)

$$C\Psi(x, s) = c\Psi(x, -s) \tag{19}$$

where $c = \gamma^5 K$ is the standard charge conjugation operator. The remarkable points are that this operation coincides with the s-time reversal operation in the Wigner sense (Galioli and Garcia Alvarez, 1995a)

$$C = S \tag{20}$$

and PCT looks like a "parity" operation in the five-dimensional space-time:

$$PcT = \gamma^5 Q \tag{21}$$

where

$$Q\Psi(x) = \Psi(-x) \tag{22}$$

and γ^5 plays the role of the "intrinsic parity" operator. The identity (20) is the quantum analogs of a celebrated Feynman (1948) observation at the classical level, that charge conjugation in the Lorentz force law is equivalent to a proper time reversal. In other words, charge conjugation is equivalent to an inversion of the sign of dx^0/ds, according to the Stueckelberg interpretation for antiparticles.

In order to get a more intuitive insight about why this proper time formalism works, let us return to the problem of particle creation in an external electromagnetic field. In this case, the Heisenberg equations of motion are

$$\frac{d\gamma^\mu}{ds} = 2i\gamma^\mu \mathcal{H} - 2i\pi^\mu \tag{23}$$

$$\frac{d\pi^\mu}{ds} = eF^{\mu\nu}\gamma_\nu \tag{24}$$

which form a coupled system of linear differential equations of first order

[11] Ironically, some years before it was Dirac himself (Dirac, 1942; see also Pauli, 1943) who introduced indefinite metric Hilbert spaces in quantum field theory with the hope of removing the true anomaly: the divergences.

in $\gamma^\mu = dx^\mu/ds$ and $\pi^\mu = p^\mu - eA^\mu$, where the mass operator $\mathcal{H} = \gamma^\mu \pi_\mu$ is a constant of motion.

Let us restrict consideration to the case of pure electric field, and choose the coordinate system in such a way that $\mathbf{E} = E\mathbf{e}_1$; therefore the only nonvanishing components of the electromagnetic field tensor are $F_{10} = -F_{01} = E$, and the system of differential equations reduced to

$$\frac{d}{ds}\begin{bmatrix}\gamma^0\\ \gamma^1\\ \pi^0\\ \pi^1\end{bmatrix} = \begin{bmatrix}2i\mathcal{H} & 0 & -2i & 0\\ 0 & 2i\mathcal{H} & 0 & -2i\\ 0 & -eE & 0 & 0\\ -eE & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\gamma^0\\ \gamma^1\\ \pi^0\\ \pi^1\end{bmatrix} \tag{25}$$

plus uncoupled equations for the components 2 and 3, identical to the free case (Barut and Zanghi, 1984; Barut and Thacker, 1985; Gaioli and Garcia Alvarez, 1996)

$$\frac{dx^\mu}{ds} = \frac{p^\mu}{\mathcal{H}} + \left[\frac{dx^\mu}{ds}(0) - \frac{p^\mu}{\mathcal{H}}\right]\cos(2ps) - \frac{1}{2p}\frac{d\gamma^\mu}{ds}(0)\sin(2ps) \tag{26}$$

The system of differential equations could be exactly solved by diagonalizing the matrix of (25). The eigenvalues are $z_{1,2,3,4} = i\mathcal{H} \pm \sqrt{-\mathcal{H}^2 \pm 2ieE}$. In the weak-field approximation ($\mathcal{H}^2 >> 2eE$) the solution of this system adopts an especially simple form (Gaioli and Garcia Alvarez, 1996)

$$\frac{dx^0}{ds}(s) = \frac{dx^0}{ds}(s)\bigg|_{E=0}\cosh\left(\frac{eE}{\mathcal{H}}s\right) - \frac{dx^1}{ds}(s)\bigg|_{E=0}\sinh\left(\frac{eE}{\mathcal{H}}s\right) \tag{27}$$

$$\frac{dx^1}{ds}(s) = \frac{dx^1}{ds}(s)\bigg|_{E=0}\cosh\left(\frac{eE}{\mathcal{H}}s\right) - \frac{dx^0}{ds}(s)\bigg|_{E=0}\sinh\left(\frac{eE}{\mathcal{H}}s\right) \tag{28}$$

where $p = \sqrt{p^\mu p_\mu}$ is the free positive mass operator. The classical picture of (26) together with (28) is a helical motion in the space, and the orbital angular momentum of this *Zitterbewegung* gives rise to the normal magnetic moment of the electron (Barut and Zanghi, 1984; Barut and Thacker, 1985; Gaioli and Garcia Alvarez, 1996). Equations (27) and (28) describe the classical hyperbolic motion derived from the Lorentz force law modulated by the free *Zitterbewegung*. This quick oscillatory motion (of the order of a Compton space-time wavelength) vanishes in the classical limit. Two different s-time scales appear, one related to the inverse of the frequency of the *Zitterbewegung* $1/(2\mathcal{H})$ and the other related to the inverse of the electric field strength $\mathcal{H}/eE$. Then when $\mathcal{H}/eE >> 1/(2\mathcal{H})$, the *Zitterbewegung* does not feel the adiabatic changes in the mean classical motion, so it works as in the free case. The same scales also appear in the space-time trajectories. If the minimal

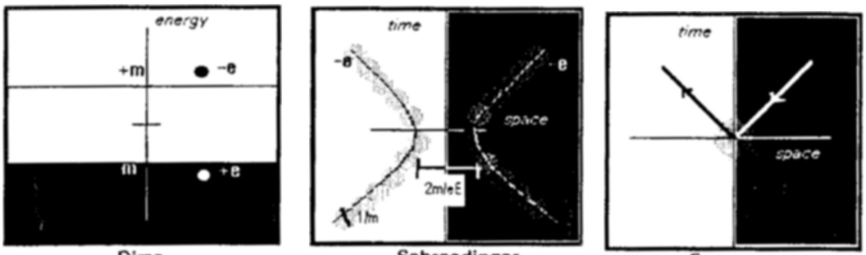

Fig. 2. Pair creation: the dark side of relativistic quantum mechanics.

distance $2\mathcal{H}/eE$ between the two branches of the hyperbola—representing particle and antiparticle solutions at the classical level—is greater than $1/\mathcal{H}$, the particle and antiparticle trajectories are distinguishable. However, when $2\mathcal{H}/eE \approx 1/\mathcal{H}$, such trajectories overlap, increasing the probability that the particle jumps to the trajectory of the antiparticle and vice versa. These jumps are reinterpreted in the standard viewpoint—which parametrizes the dynamics with the coordinate time x^0—as the pair creation and annihilation processes (Dirac picture[12]). Summarizing, the Schrödinger *Zitterbewegung* depicted above gives a very clear semiclassical interpretation of such processes, which dresses the corresponding Feynman diagrams in physical content, disregarding the concept of the Dirac sea (see Fig. 2).

At this point we disagree with some field theorists who regard Feynman's graphical method as "a convenient pictorial device that enables us to keep track of the various terms in the matrix elements which can be rigorously derived from quantum field theory" (Sakurai, 1967, p. 241; see also comments by Dyson cited on the same page). We think that they do not completely take into account the genesis of Feynman's ideas originally developed from the proper time method. Unfortunately, due to the misunderstanding of his dissertation at Pocono, (Schweber, 1986), Feynman was forced to introduce his space-time visualization of quantum electrodynamic processes in the form written in Feynman (1949), relegating much of his original physical ideas and motivations to Feynman (1950, 1951). So there is a generation of field theorists who learned the derivation of the Feynman rules from Dyson (1949) rather than from Feynman's papers. In fact, when Dyson's paper appeared most of Feynman's work was still unpublished. Unfortunately although Dyson himself remarked that "the theory of Feynman differs profoundly from that of Schwinger and Tomonaga," the announcement of the demonstration of the equivalence (strictly speaking only at the level of the consequences) of

[12]This picture was refined by Sauter by considering the deformation of the energy gap produced by the electric field. Pair creation is interpreted as a tunneling of a negative-energy state (not a hole in a sea) to a positive-energy state (Lifshitz and Pitaevskii, 1971).

both theories had great impact. Moreover, the fine Schwinger (1951) calculations using a proper time method were considered just as mathematical tools and Nambu's claims of his deep paper of 1950:

> The space-time approach to quantum electrodynamics, as has been developed by Feynman, seems to offer a very attractive and useful idea to this domain of physics. His ingenious method is indeed attractive, not only because of its intuitive procedure which enables one to picture to oneself the complicated interactions of elementary particles, its ease and relativistic correctness with which one can calculate the necessary matrix elements or transition probabilities, but also because of its way of thinking which seems somewhat strange at first look and resists our minds that are accustomed to causal laws. According to the new standpoint, one looks upon the world in its four-dimensional entirety. A phenomenon that will come into play in this theatre is now laid out beforehand in full detail from immemorial past to ultimate future and one investigates the whole of it at glance. The time itself loses sense as the indicator of the development of phenomena; there are particles which flow down as well as up the stream of time; the eventual creation and annihilation of pairs that may occur now and then, is no creation nor annihilation, but only a change of directions of moving particles, from past to future, or from future to past; a virtual pair, which, according to the ordinary view, is foredoomed to exist only for a limited interval of time, may also be regarded as a single particle that is circulating round a closed orbit in the four-dimensional theatre; a real particle is then a particle whose orbit is not closed but reaches to infinity . . .

received little attention.

On the other hand, most quantum field theory treatises which attempt to incorporate the Feynman space-time visualization turn out to be contradictory. For example, they interpret field operators as operators that create and annihilate particles at space-time points in order to give an interpretation to the Green functions. However, relativistic and nonrelativistic quantum fields exhibit a striking difference concerning the localizability of their respective field quanta (Lurié, 1968). In fact, while in the nonrelativistic case there is in principle no limitation on the accuracy of measuring the position of a particle, the combination of relativity and quantum theory provides an intrinsic limitation to the measurability of the position due to the particle creation mechanism. The understanding of such difficulties has inclined some authors to propose the idea that Minkowski space-time is not suitable for particle physics and its role was essentially a historical one,[13] unlike the energy-momentum space, which would be fundamental (Bacry, 1988). On the con-

[13] Although this hypothesis could work for the Poincaré group in the case of free fields, strong difficulties arise upon introducing interactions. Bear in mind that localizability and minimal coupling are intimately linked. Moreover, this fact is not compatible with the principle of general covariance. Notice that it would be possible to extend this formulation to develop quantum field theory in curved space-time.

trary, in our proposal we prefer to leave the Poincaré group and retain the localizability in Minkowski space-time.

Summarizing, those field theories which desire to keep the interpretive picture of the Feynman diagrams must give up the Poincaré group. There is no space-time localization of particles in this framework. There is only space-time localization of charges off the mass shell.

In order to reinforce the pictorial image of the Fig. 2, let us derive the one-loop effective action $W^{(1)}$, which describes the pair creation in an external electromagnetic field, from an argument purely based on the proper time formalism. As $W^{(1)}$ is i times the closed-loop amplitude L, let us compute L using the proper time formalism. First, let us evaluate the amplitude for a super-particle at x^μ and polarization k at time $s = 0$, remaining at the same point and with the same polarization at time s. As a consequence of the indefinite metric (17), the spectral resolution of the identity is

$$I = \int d^4x \sum_{jk} \gamma^0_{jk} |j, x^\mu\rangle\langle k, x^\mu| \tag{29}$$

Then the expression of such an amplitude per unit of proper time for all the degrees of polarization is $(1/s)\Sigma_{jk}\, \gamma^0_{jk}\langle k, x^\mu| e^{i(\gamma^\mu \pi_\mu)s} | j, x^\mu\rangle$. The above process is represented through an open diagram in the five-dimensional space-time, but it is a closed loop in four dimensions (Davidon, 1955a, b). Restricting the formalism to the mass shell by means of a Fourier transformation in proper time with the causal prescription and summing the contributions of each space-time point, we finally have

$$W^{(1)} = i \int \int_0^\infty \frac{1}{s} \sum_{jk} \gamma^0_{jk} \langle k, x^\mu | e^{i(\gamma^\mu \pi_\mu)s} | j, x^\mu\rangle e^{-ims} ds\, d^4x \tag{30}$$

Schwinger, using quantum field theory, obtained (30), which became the starting point of his seminal paper (Schwinger, 1951; Feynman, 1950, 1951).

The procedure used in the calculation of $W^{(1)}$ also shows that the ultraviolet divergences only appear after the reduction of the off-shell amplitude on the mass shell. Note that this circumstance also suggests a natural regularization method based on a small mass dispersion (Feynman, 1950, 1951). Our alternative explanation does not involve the infinite amount of energy and charge of the Dirac sea in order to consider antiparticles, and in this way it avoids the infinities introduced in the standard theory from the very beginning. This is the reason why closed loops do not appear in the off-shell theory.

Until now we have only discussed the theory of external fields. In order to conclude, let us briefly discuss the radiative process.

Using this formalism and his operator calculus, Feynman presented at Pocono a closed expression for a system of spin-half charges interacting via the quantized electromagnetic field for the case in which only virtual photons are present. In the particular case of one charge it reads (Feynman, 1950, 1951; Schweber, 1986)

$$\Psi(x, s) = \exp\left\{-i\left[\int_0^s \gamma^\mu(s')\pi_\mu(s')\,ds' + e^2\int_0^s\int_0^s \gamma^\mu(s')\gamma_\mu(s'')\delta_+\{[x_\mu(s') - x_\mu(s'')]^2\}\,ds'\,ds''\right]\right\}\Psi(x, 0) \tag{31}$$

where $\delta_+\{[x_\mu(s') - x_\mu(s'')]^2\}$ is the Green function of the d'Alembertian with Feynman's boundary conditions. From the second term (31) Feynman showed that the radiative corrections of QED can be derived. The analogy between the phase of (31) and the Wheeler–Feynman action (Wheeler and Feynman, 1945; Feynman, 1948) for classical electrodynamics is remarkable. In fact the only substantial difference is in the boundary conditions (half-advanced and half-retarded) chosen for the d'Alembertian Green function. The right boundary conditions for QED can be obtained from the retarded condition of the off-shell theory. This fact strongly suggests that (31) could be derived from first principles from a de Sitter-invariant formulation of QED.

For one super-particle (antiparticle) the de Sitter-invariant equations read

$$\Gamma^A(i\partial_A - eA_A)\Psi = 0 \tag{32}$$

$$\partial_A F^{AB} = e\bar{\Psi}\Gamma^B\Psi \tag{33}$$

where the super-potential $A^A = (A^\mu, A^5)$ arises from a natural extension of the gauge principle (Shnerb and Horwitz, 1993). The standard four-potential can be obtained from A^A, integrating the first four components in the proper time

$$A^\mu(x^\nu) = \int_{-\infty}^{+\infty} A^\mu(x^\nu, s)\,ds$$

as in the case of the matter fields. [The exponential factor does not appear in this case because the photon is nonmassive. Note also that the transformation $A^\mu(x^\nu, s) \to A^\mu(x^\nu, -s)$, $(ds \to -ds)$, leads to the standard notion of charge conjugation for the potentials.]

NOTE ADDED IN PROOF

After completing this work we discovered a review paper of Fanchi (1993) and the closely related works of Herdegen (1982) and Kubo (1985).

ACKNOWLEDGMENTS

We are grateful to the organizers of the First International Colloquium on Actual Problems in Quantum Mechanics, Cosmology, and the Primordial Universe and the Foyer d'Humanisme for their warm hospitality.

We want to express our thanks to Juan León García, Lawrence Horwitz, and Marek Czachor for their encouragement.

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